Advanced and Contemporary Topics in Macroeconomics I

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Class Lecture Note 4 _ Chapter 4

Real Business-Cycle Theory

Addis Ababa University

(Since 1950)

Chapter 4
Real Business-Cycle Theory

Some basic Facts about Economic Fluctuations:

1. Aggregate fluctuations do not follow any regular or cyclical patterns but various types of disturbances affecting output and employment seem to occur randomly
2. Fluctuations are unevenly distributed over different components of output.
3. Output growth is fairly symmetrically distributed around its mean but there is an asymmetry in the length of booms and recessions.
4. The magnitude of fluctuations differ substantially between different time periods.

Real Business-Cycle Theory

Point of departure is a baseline Walrasian model as the Ramsey model in Chapter 2.

- -As in Ramsey model, this model excludes all externalities and the possibility of heterogeneous households.
- -To incorporate the possibility of aggregate fluctuations this type of model must be extended in two ways (Nb: The Ramsey framework –which is the micro base of Walarasian -without shocks, converges to balanced grwoth path). So we will introduce 2 shocks:
  - (1)There must be some sources of disturbances: this is taken to be technology/production function or government purchase (in more recent literature) . These are real shocks and hence the name Real Business Cycle (RBC) models
  - (2) It must be possible for household to vary their labor supply, i.e. there must be variations in employment

A Baseline RBC Model

- The economy consists of:
  - A large number of identical price-taking firms
  - A large number of identical price-taking households that are infinitely lived
- There are two production factors, K and L
- The production function is Cobb-Douglas with CRS

$$Y = K_t^{\alpha}(AL)_t^{1-\alpha}{}_t \qquad 0 < \alpha < 1$$

- Labor and capital is paid their marginal products:

$$w_t = (1-\alpha)k^{\alpha}A_t \qquad r_t = \alpha k^{\alpha-1} - \delta$$

- [3] [4]

A Baseline model (cont.)

- Output is divided between consumption (C), investment (I) and government expenditure (G).
- The capital stock in period t+1 is then given by:
- $$\begin{aligned} K_{t+1} &= K_t + I_t - \partial K_t \\ &= K_t + Y_t - C_t - G_t - \partial K_t \end{aligned} \qquad [2]$$

- The government purchases are financed by lump-sum taxes and the budget is assumed to be balanced in each period.

A Baseline model (cont.)

- The representative household maximizes the expected value of

$$U = \sum_{t=0}^{\infty} e^{-\rho t} u(c_t, 1-\ell_t) \frac{N_t}{H} \qquad [5]$$

- Where $u(c_t, 1-\ell_t)$ is the instantaneous utility function of the individual household member, which is a function of consumption and leisure.
- For simplicity it is assumed that *u* is log-linear in its two argument such that:
- $$u_t = \ln c_t + b \ln(1-\ell_t), \qquad b > 0 \qquad [6]$$

A Baseline model (cont.)

N is the size of the population, H the number o f households; so N_t /H is the size of the household [members of hh] in period t.

- The population is assumed to grow at rate n:
- $$\ln N_t = \overline{N} + nt, \qquad n < \rho \qquad [7]$$
- Technology is assumed to grow according to an auto-regressive process:

$$\ln A_t = \overline{A} + gt + \tilde{A}_t \qquad \text{where } \tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}$$

$$\ln A_t = \overline{A} + gt + \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t} \qquad -1 < \rho_A < 1 \qquad [8]$$

- Government expenditure is assumed to grow according to:

$$\ln G_t = \overline{G} + (n+g)t + \rho_G \tilde{G}_{t-1} + \varepsilon_{G,t} \qquad -1 < \rho_G < 1$$

Household behavior: Inter-temporal Substitution in Labor Supply

- Assume that households only live for only one period and that there is only one household member so that the Langrangian simplifies to:

$$\mathcal{L} = \ln c_t + b \ln(1 - \ell_t) + \lambda(w\ell - c)$$

First-order conditions for c and **l,** respectively, are**

$$\frac{1}{c} - \lambda = 0 \qquad -\frac{b}{1-\ell} + \lambda w = 0 \quad \rightarrow \quad -\frac{b}{1-\ell} + \frac{1}{\ell} = 0$$

Which implies that the labor supply is independent of the wage.

Inter-temporal substitution in labor supply…. (cont.)

Assume that households live for two periods and that there is only one household member. The Langrangian becomes:

$$\mathcal{L} = \ln c_1 + b\ln(1-\ell_1) + e^{-\rho}\left[\ln c_2 + b\ln(1-\ell_2)\right]$$
$$+\lambda\left[w_1\ell_1 + \frac{1}{1+r}w_2\ell_2 - c_1 - \frac{1}{1+r}c_2\right]$$

First-order conditions for **l_1** and **l_2**, respectively, are

$$\frac{b}{1-\ell_1} = \lambda w_1 \qquad \frac{e^{-\rho}b}{1-\ell_2} = \frac{1}{1+r}\lambda w_2$$

Combining these conditions, using λ, yields:

$$\frac{1-\ell_1}{1-\ell_2} = \frac{1}{e^{-\rho}(1+r)}\frac{w_2}{w_1}$$

Inter-temporal substitution in labor supply…. (cont.)

$$\frac{1-\ell_1}{1-\ell_2} = \frac{1}{e^{-\rho}\left(1+r\right)}\frac{w_2}{w_1}$$

The condition for inter-temporal substitution in labor supply, implies that:

- A rise in w_1 relative to w_2 results in a decrease in period 1 leisure [$1-l_1$] and vice verse.
- A rise in r rises first period labor supply relative to second period labor supply.

Household behavior under Uncertainty

- Because of uncertainty about rates of return to capital and labor, households do not choose deterministic paths for consumption and labor supply.
- Instead, household choices in each period depend on expectations (which are likely to be influenced by shocks in technology and government spending up to that date).
- It is possible to describe household behavior informally [instead of solving the full optimization as in RCK model] by investigating the effects of marginal changes in different variables in one period.
  - If we reduce consumption by ΔC and the use the resulting wealth to increase C in the next period above what it otherwise would be; then
  - If households are behaving optimally, marginal changes of this type in one period should leave the expected utility over the coming periods unchanged

Household behavior under uncertainty (cont.)

- Consider a marginal reduction in current consumption per household member in period t. The marginal utility of this change is

$$\frac{\partial U}{\partial c_t} = e^{-\rho t} \frac{N_t}{H} \frac{1}{c_t}$$

which implies that the utility cost of a change in c is: $e^{-\rho t}(N_t / H)(\Delta c / c_t)$

Since hh has e^n times as many members in period *t+1* as in period *t*, the increased in consumption per member in *t+1* is $e^{-n}(1 + r_{t+1})\Delta c$

The marginal utility of consumption at *t+1* per member is

$$e^{-\rho(t+1)}(N_{t+1} / H)e^{-n}(1 / c_{t+1})$$

Thus the expected utility benefit as of period *t* is:

$$E_t\left[e^{-\rho(t+1)}(N_{t+1} / H)e^{-n}(1 + r_{t+1}) / c_{t+1}\right]\Delta c$$

Household behavior under uncertainty (cont.)

Equating the utility cost with expected utility benefit gives:

$$e^{-\rho t}\frac{N_t}{H}\frac{\Delta c}{c_t}=E_t\left[e^{-\rho(t+1)}\frac{N_{t+1}}{H}e^{-n}\frac{1}{c_{t+1}}\left(1+r_{t+1}\right)\right]\Delta c$$

Since the growth in household size is not uncertain and $N_{t+1}=N_t\, e^{n}$ this condition can be simplified to

$$\frac{1}{c_t}=e^{-\rho}E_t\left[\frac{1}{c_{t+1}}\left(1+r_{t+1}\right)\right] \qquad [23]$$

This condition for optimal household consumption implies that the trade off between present and future consumption depends both on the expectations of future marginal utility $[1/c_{t+1}]$ and rate of returns $(1+r_{t+1})$ but also on their interaction. Hence, the equation above implies:

$$\frac{1}{c_t}=e^{-\rho}\left\{E_t\left[\frac{1}{c_{t+1}}\right]E_t\left(1+r_{t+1}\right)+\mathrm{cov}\left(\frac{1}{c_{t+1}},1+r_{t+1}\right)\right\}$$

The tradeoff between consumption and labor supply

At each point in time the household choose not only consumption but also labor supply [Using Eqns [5] and [6]].

The 'disutility' of a marginal change in labor supply is:

$$\frac{\partial U}{\partial \ell_t} \Delta \ell = e^{-\rho t} \frac{N_t}{H} \frac{b}{1-\ell_t} \Delta \ell$$

The marginal benefit of a marginal change in labor supply is

$$\frac{\partial U}{\partial c_t} w \Delta \ell = e^{-\rho t} \frac{N_t}{H} \frac{1}{c_t} w_t \Delta \ell$$

The tradeoff between…. (cont.)

Equating the cost in utility from an increase in labor supply with the utility benefit of this increase equals:

$$e^{-\rho t}\frac{N_t}{H}\frac{b}{1-\ell_t}\Delta\ell = e^{-\rho t}\frac{N_t}{H}\frac{1}{c_t}w_t\Delta\ell$$

$$or \qquad \frac{c_t}{1-\ell_t} = \frac{w_t}{b} \qquad [26]$$

which relates current consumption & leisure, given the current wage rate.

- Note that the tradeoff between consumption and labor supply in a given period only depends on current period variables, so it does not involve any uncertainty.
- This completes the RBC baseline model.

Solving the Model

The baseline RBC model noted cannot be solved analytically without changes, because it contains both linear and log-linear [such as the PF &U] components. Two possible solutions:

- (1) Log-linearization of the model by replacing the linear components (e.g. growth in technology and government) with first order Taylor approximations in the logs of the relevant variables around the path that the economy would follow in the absence of shocks.
- (2) Simplify the model by assuming that there is no government and assuming a depreciation rate of 100 %, implying that the capital stock [Eqn 2] and the real interest rate [Eqn 4] is given by:

$$K_{t+1} = Y_t - C \qquad 1 + r_t = \alpha k^{\alpha - 1}$$

Solving the model (cont.)

- With perfectly competitive markets, no externalities and a finite number of individuals, the model's equilibrium must be Pareto efficient. Hence, the model can be solved by finding the competitive equilibrium.
- The equilibrium is found where the two endogenous variables, consumption (or saving) and labor supply, satisfy the conditions for:
  - household optimization
  - growth in capital
  - real interest rate in competitive markets.
- We focus on condition for hh optimization [Eqns 23 &26] for other equations follow mechanically from accounting and from competetion. You do that!.

Solving the model (cont.)

Writing the model in log-linear form and making use of the following expressions [from Eqn 23]:**

$$c_t = (1-s)Y_t / N_t$$

$$K_{t+1} = Y_t - C_t = s_t Y_t$$

$$1 + r_t = \alpha k^{\alpha-1}$$

implies that the condition for optimal household consumption [Eqn 23] can be written as:

$$-\ln(1-s_t) - \ln Y_t + \ln N_t = -\rho + \ln E_t\left[\frac{\alpha Y_{t+1}}{K_{t+1}(1-s_{t+1})Y_{t+1} / N_{t+1}}\right]$$

$$= -\rho + \ln E_t\left[\frac{\alpha N_{t+1}}{s_t(1-s_{t+1})Y_t}\right]$$

$$= -\rho + \ln\alpha + \ln N_t + n - \ln s_t - \ln Y_t + \ln E_t\left[\frac{1}{(1-s_{t+1})}\right]$$

Solving the model (cont.)

Since the values of α, N_t, n, s_t, Y_t are known at date t we can write:

$$-\ln(1-s_t)-\ln Y_t+\ln N_t=-\rho+\ln\alpha+\ln N_t+n-\ln s_t-\ln Y_t+\ln E_t\left[\frac{1}{(1-s_{t+1})}\right]$$

which simplifies to:

$$\ln s_t-\ln(1-s_t)=-\rho+n+\ln\alpha+\ln E_t\left[\frac{1}{(1-s_{t+1})}\right]$$

NB: Crucially the two state variables (K and A) do not enter in this equation, implying that there is a constant value of s, which satisfies the optimization problem of the representative household .

Solving the model (cont.)

If s is constant at some value $\hat{s}$ then s_{t+1} is not uncertain and so $E_t[1/(1-s_{t+1})]$ is simply $1/(1-\hat{s})$. Thus the above equations (which is , the condition for household optimization with a constant saving rate, $\hat{s}$ (under the assumption of 100 % depreciation) simplifies to:

$$\ln \hat{s} = \ln \alpha + n - \rho$$

$$\rightarrow \quad \hat{s} = \alpha e^{n-\rho}$$

Thus, the saving rate is constant.

Solving the model (cont.)

The household does not only choose consumption but also labor supply, according to the condition that [see Eqn 26]

$$\frac{c_t}{1-\ell_t} = \frac{w_t}{b}$$

Making use of the fact that $c_t = (1-\hat{s})Y_t / N_t$, we have:

$$\ln\left[(1-\hat{s})\frac{Y_t}{N_t}\right] - \ln(1-\ell_t) = \ln w_t - \ln b$$

Solving the model (cont.)

With the assumed production function (Cobb-Douglas) the wage rate in a competitive equilibrium is given by

$$w_t = (1-\alpha)Y_t / \ell N_t$$

Substuting this in the above equation yields:

$$\ln(1-\hat{s}) + \ln Y_t - \ln N_t - \ln(1-\ell_t)$$

$$= \ln(1-\alpha) + \ln Y_t - \ln \ell_t - \ln N_t - \ln b$$

$$\Rightarrow \ln \ell_t - \ln(1-\ell_t) = \ln(1-\alpha) - \ln(1-\hat{s}) - \ln b$$

Which in turn equals**

$$\ell_t = \frac{1-\alpha}{(1-\alpha) + b(1-\hat{s})} \qquad [37]$$

Solving the model (cont.)

:

$$\ell_t = \frac{1-\alpha}{(1-\alpha)+b(1-\hat{s})}$$

- This result shows that the labor supply does not depend on the state variables (A and K) implying that labor supply is also constant in a competitive equilibrium.
- The remainign equation of the model do not involve optimization (they forllow form technology, accounting and competion). Thus we have found a solution for the model with constant *s* and *l*

Solving the model (cont.)

The reason for a constant labor supply in spite of the possibility of inter-temporal substitution is that movements in capital or technology have an offsetting impact on the impact of "the relative wage and the interest rate effects" on the labor supply:

- ⇨ Eg. An improvement in technology raises the current wage relative to the future wage, which stimulates current labor supply. But the higher wages by raising the amount saved (despite the constant saving rate) results in a lower expected interest rate, which reduces labor supply.

In the specific case we have considered here these two effects exactly balance each other.

Implications of the model

- The model provides an example of the economy where real shocks (variation in capital accumulation and technological progress) causes output fluctuations.
- Since markets are perfect/Walrasina the movements in output is the optimal response to shocks.
- The RBC models main implications is thus: "Contrary to conventional wisdom, macroeconomic fluctuations do not reflect any market failure, reflects agents optimal response. Hence any government intervention reduces welfare"
  - Not this is an interesting result which is contrary to Kenyenisan macroeconomics and in fact the practice of the West during the 2008/09 financial-cum-economic crisis! It is also idelogical!

Implications of the model

- With a Cobb-Douglas production function (with CRS and Harrod-neutral labor augmenting technology) it is readily shown that output movements follow a second-order auto-regressive process

  i.e. deviation of output from its long run trend is a linear combination of its two previous values plus a white-noise disturbance.

- We will show this using the RBC model we just have developed.

Implications of the model (cont.)

Combining the following equations from the base-line model:

$$\ln Y_t = \alpha \ln K_t + (1-\alpha)(\ln A_t + \ln L_t)$$

$$K_t = \hat{s} Y_{t-1}, \qquad L_t = \hat{\ell} N_t$$

$$\ln N_t = \bar{N} + nt, \quad \ln A_t = \bar{A} + gt + \tilde{A}_t$$

results in the following expression for the log of output:

$$\ln Y_t = \alpha \ln \hat{s} + \alpha \ln Y_{t-1} + (1-\alpha)(\ln \bar{A}_t + gt) + (1-\alpha)\tilde{A}_t + (1-\alpha)(\ln \hat{\ell} + \bar{N} + nt) \qquad [39]$$

Implications of the model (cont.)

The only two components of the equation [39] giving the log of output in period t that do not follow deterministic paths are

$\alpha \ln Y_{t-1}$ and $(1-\alpha)\tilde{A}_t$.

We can therefore write $\tilde{Y}_t$ as the deviation of $\ln Y_t$ from the path it would follow if there were no shocks in technology and hence $\ln A_t = \bar{A} + gt$, thus Eqn 39 reduces to:

$$\tilde{Y}_t = \alpha \tilde{Y}_{t-1} + (1-\alpha)\tilde{A}_t \quad [40]$$

Since [40] holds for each period we have

$$\tilde{Y}_{t-1} = \alpha \tilde{Y}_{t-2} + (1-\alpha)\tilde{A}_{t-1}$$

$$\tilde{A}_{t-1} = \frac{1}{1-\alpha}\left(\tilde{Y}_{t-1} - \tilde{Y}_{t-2}\right) \quad [41]$$

Implications of the model (cont.)

- With A_t following a first-order auto-regressive process [see Eqns 7&8],

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}$$

- Substituting the fact above and Eq [41] into Eqn [40] we obtain:

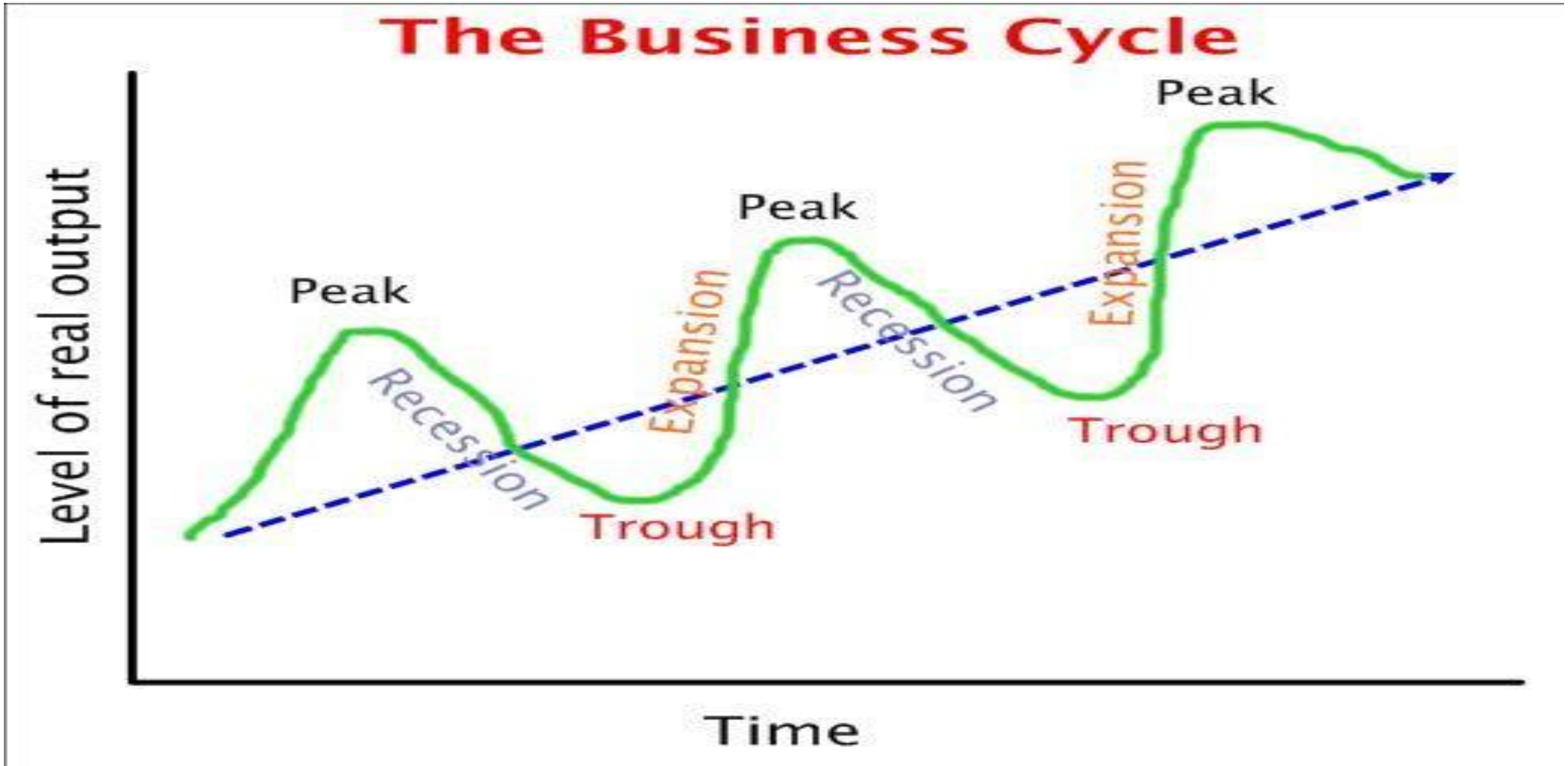
$$\begin{aligned} \tilde{Y}_t &= \alpha \tilde{Y}_{t-1} + (1-\alpha)\left(\rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}\right) \\ &= \alpha \tilde{Y}_{t-1} + \rho_A\left(\alpha \tilde{Y}_{t-2} - \alpha \tilde{Y}_{t-1}\right)(1-\alpha)\varepsilon_{A,t} \\ &= (\alpha + \rho_A)\tilde{Y}_{t-1} - \alpha\rho_A \tilde{Y}_{t-2} + (1-\alpha)\varepsilon_{A,t} \end{aligned}$$

- We note here departure of log output form its normal path follows a 2nd order distributed lag process (a linear combination of past 2 years values and a white noise process)
- Observe that the combination of a positive coefficient on the first lag and a negative coefficient on the second lag can cause output to have hump-shaped [typcl cycle graph] responses to real shocks.

Implications of the model (cont.)

- (see the graph reproduce below)

- Because of a relative small size of α , output dynamics is largely driven by the persistence in technology shocks, ρ_A The value of this parameter must be relatively high for technology to have long run impacts on output.

- 

How well does the model fit with real world data in the West?

- Data on the U.S and our RBC model shows that:
  - If actual US log output is detrended linearly it follows a process similar to hump-shaped model described here
  - In practice investment fluctuates more than consumption, while in this model owing to constant saving rates consumption and investment are equally volatile
  - In the model labour input doesn't vary, in practice, however employment and hours worked out are strongly pro-cyclical,.
  - The model predicts a one-to-one relationship between real wages and output. In practice the relationship is moderate.

How well does the model fit….Con’td

- If we solve a more general version of the model that assumes incomplete depreciation and include a government sector imposing taxes, the model fits considerably better with real world data:
  - With less than complete depreciation the inter-temporal substitution effect is more important, implying that a positive technology shock stimulate both labor supply and increases saving rates.
  - With a less than complete depreciation, technology shocks will not result in ‘offsetting effects of real wages and interest rates’ on labor supply. Instead some of the adjustments of the capital stock occur through changes in labor supply rather than changes in savings.
  - Introducing shocks to government purchase [taxation etc] into the model breaks the strong link between real wages and output [for instance increase gov’t purchase…increase life time tax liability…leads to reduce life time wealth of hhs and hence they reduce leisure / work more].

How well does the model fit….Con’td

Calibrations of variants of the base-line real business-cycle model on U.S. data by Prescott (1986) and Hansen (1985) (among others) find that:

- The model works well in predicting fluctuations in aggregate output
- The model is consistent with real world data in predicting that consumption is considerably less volatile than investments
- However, the model fails to predict the contributions of variation in labor supply to variations in output.
- Monetary shocks cannot be analyzed with this type of model since the money supply have no effect on technology, preferences, depreciation rates or government expenditure!
  - Monetary shocks have only nominal effects in models where prices are completely flexible
  - A lot of research indicate that monetary shocks have real effects because of nominal rigidity [Specially see Austrian school: Meis, Schumpeter etc where money and monetary issues are central in business cycles what about the recent financial-economics crise 2008/09]

Limitations of the model (cont.)

- The model emphasize the importance of technology shocks but in reality such disturbances are difficult to identify. Attempts to do so (Shea, 1998; Francis and Ramey, 2005; Fernald, 2007; among others) show that technological shocks tend to lower labor input while output is rising.
  - This outcome is consistent with the predictions about economic adjustment when prices are sticky:
  - in the short run output rises because of a positive demand shock whereas the labor needed to produce this output falls due to an increase in labor productivity.
- As we noted earlier neglect of monetary/money issues is a major weakness of these models.

Further Reading /Assignment: Michael Kalecki"Dynamics of the Capitalist Economy. Karl Marx "Capital, vol I & II" ;Schumpeter "Various Issues on Economic Fluctuation" will give you completely a different perspective on business cycles./END/